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Source: *Econometrica*, Vol. 46, No. 4 (Jul., 1978), pp. 807-816
Published by: The Econometric Society
Stable URL: http://www.jstor.org/stable/1909750
Accessed: 20/01/2011 10:16

Econometrica, Vol. 46, No. 4 (July, 1978)

INFORMATION, EFFICIENCY, AND THE CORE OF AN ECONOMY[1]

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The meaning of exchange efficiency is examined in the context of an economy in which agents differ in their endowments of information. Definitions of efficiency, and of the core, are proposed which emphasize the role of communication. Opportunities for insurance are preserved by restricting communication, or in a market system by restricting insider trading, prior to the pooling of information for the purposes of production.

MY SUBJECT IS AN ECONOMY in which different agents have different information. I propose a definition of exchange efficiency and I characterize the efficient allocations. I then examine an analogous definition of the core and I demonstrate that the core is not empty if the usual regularity conditions are satisfied. An example, however, illustrates that a market process may fail to yield an efficient allocation. In fact, in this example the market allocation is not even individually rational for the agents. Also, in this example the core is empty if there are opportunities for communication which disrupt arrangements for mutual insurance.

1. FORMULATION

S denotes the set of possible states. For simplicity I suppose that the cardinality of S is finite. Some one state s^* in S is the prevailing state. An event is a subset of S.

N denotes the finite set of agents. The information of the ith agent is described by the field F_i of events which he can discern. An event E is in the field F_i iff he knows whether the prevailing state is in the event E or in the complementary event $S\backslash E$. For example, if agent i observes the value of a random variable y_i, then F_i is the smallest field containing the events in the inverse image of y_i. The minimal nonempty events in the field F_i form a partition of the states denoted by PF_i. Precisely one member of the partition is known by the agent to contain the prevailing state. $PF_i(s)$ denotes the unique member of the partition containing the state s.

The field of events discernable by every agent is the "coarse" field $\bigwedge_N F = \bigcap_{i \in N} F_i$. By pooling their information they could discern the events in the "fine" field $\bigvee_N F$ for which $P\bigvee_N F(s) = \bigcap_{i \in N} PF_i(s)$. More generally, the result of a communication system (c.s.) is a collection $(H_i)_{i \in N}$ of fields such that $H_i \supseteq F_i$ for each agent and $\bigvee_N H = \bigvee_N F$. Communication enlarges the field of events an agent can discern but it does not produce new information. The null c.s. is $(F_i)_{i \in N}$ and the full c.s. is $(\bigvee_N F)_{i \in N}$.

[1] This study was supported in part by the National Science Foundation, Grant SOC76-11446-A01, at the Institute for Mathematical Studies in the Social Sciences, and in part by the Energy Research and Development Administration, Contract E(04-3)-326 PA # 18, at the Systems Optimization Laboratory, Stanford University.

[2] For discussions and suggestions I am indebted to Robert Aumann, Jonathan Cave, Bengt Holmström, Takao Kobayashi, David Kreps, and John Riley. Kobayashi [3] presents extensions of this work.

A commodity bundle is a member of the Euclidean space with coordinates indexed by the commodities. An agent i has for each state s a set $X_i(s)$ of commodity bundles which are feasible for consumption. One member of $X_i(s)$ is agent i's endowment $e_i(s)$ which he obtains if s is the prevailing state and he engages in no trade. A consequence of trade is an allocation $x=(x_i(s))$ which provides agent i with the consumption $x_i(s)\in X_i(s)$ if state s prevails, provided that $\Sigma_{i\in N} x_i(s)=\Sigma_{i\in N} e_i(s)$. One may also require that the consumption plan x_i is measurable with respect to a field F_i'', namely $x_i(s)=x_i(\bar{s})$ if $s\in PF_i''(\bar{s})$. In this case I assume that X_i and e_i are F_i'-measurable for some field $F_i'\supseteq F_i$ and that $F_i''\supseteq \bigvee_N F'$.

When agent i knows that the prevailing state is in the event $A\in PF_i$ of his partition, or a finer event $A\in PH_i$ discernable from communication, he has a relation $\succ_{iA}$ of preference between feasible consumption plans. For any coarser event $E\in H_i$ the relation $x_i\succ_{iE}\bar{x}_i$ means that $x_i\succ_{iA}\bar{x}_i$ for every event $A\in PH_i$ in the partition for which $A\subseteq E$. It will suffice here to assume that this preference relation is represented by a probability assessment (S, F_i'', μ_i) and by an F_i'-measurable utility function u_i which assigns to each feasible consumption $x_i(s)\in X_i(s)$ in state s a real value $u_i(s, x_i(s))\equiv u_i[x_i](s)$. If H is a subfield of F_i'' then the conditional expectation of an F_i''-measurable random variable u defined on S is an H-measurable random variable $v\equiv \mathscr{E}_i\{u|H\}$ for which $\int_E u(s)\,d\mu_i(s)=\int_E v(s)\,d\mu_i(s)$ for each event $E\in H$. In particular, $x_i\succ_{iE}\bar{x}_i$ for an event $E\in H_i$ iff $\mathscr{E}_i\{u_i[x_i]|H_i\}(s)>\mathscr{E}_i\{u_i[\bar{x}_i]|H_i\}(s)$ for each state $s\in E$. Note that the conditional expectation has a common value $\mathscr{E}_i\{u_i[x_i]|H_i\}(A)$ for $s\in A\in PH_i$ if $\mu_i(A)>0$. For simplicity I assume that the measure μ_i assesses a positive probability for each nonempty event in $\bigvee_N$ F.

For the propositions in Sections 2–4 I impose the usual regularity assumptions which ensure that the sets of feasible allocations and attainable utilities are compact and convex. Namely, for each agent i and each state s the set $X_i(s)$ of feasible consumptions is closed, convex, and bounded below; and the utility function $u_i(s,\cdot)$ defined on this set is continuous and concave.

2. EFFICIENCY

It is useful to recognize that no single definition of efficiency will suffice for all purposes. The fact that different agents have different information must necessarily eliminate some of the opportunities for mutual insurance. Moreover, the possibility of communication raises the prospect that additional opportunities will be eliminated. My aim here is to identify that definition of efficiency which retains the greatest opportunities for insurance subject to the limitation inherent in the agents' information. In addition, I seek a definition of efficiency which is consistent with a viable definition of the core.

Two simple examples illustrate the primary considerations. There is a single desired commodity; each agent has a utility function which is independent of the state and strictly concave, reflecting aversion to risk; and each agent assigns equal probabilities to the states.

EXAMPLE 1: There are two agents and two states. The agents' endowments and information are displayed in Table I. Also shown is an allocation x which would be a favorable arrangement for mutual insurance in the absence of a difference in information. As it is, however, agent 1 has superior information. If the prevailing state is $s^* = a$ he would surely reject the proposed allocation x. That is, the allocation is not individually rational for agent 1, nor is any other

TABLE I

Agent (i)	PF_i	State (s):	Endowments ($e_i(s)$) a	b	Allocation ($x_i(s)$) a	b
1	$\{a\}, \{b\}$		2	0	1	1
2	$\{a, b\}$		0	2	1	1

allocation which partially insures agent 2 against his perceived risk. Indeed, realizing this, agent 2 has no incentive to offer or accept a contract since it could be advantageous to agent 1 only in state b when it is to his own disadvantage. I conclude that a useful definition of efficiency must include the endowment as an efficient outcome. This example illustrates the phenomenon of adverse selection which often vitiates opportunities for insurance.

EXAMPLE 2: There are three agents and three states. The agents' endowments and information are displayed in Table II. As in the previous example there is an allocation which provides an equal amount (3 units) to each agent in each state

TABLE II

Agent i	PF_i	State (s):	Endowments ($e_i(s)$) a	b	c	Allocation ($x_i(s)$) a	b	c
1	$\{a\}, \{b, c\}$		5	1	3	5	2	2
2	$\{b\}, \{a, c\}$		3	5	1	2	5	2
3	$\{c\}, \{a, b\}$		1	3	5	2	2	5

but which in each state is not individually rational for the agent with superior information. Also shown in Table II is an allocation x which escapes this feature and which provides complete insurance for the poorly informed agents. If the prevailing state is $s^* = a$, then agent 2 or 3 perceives equal chances that his endowment is 1 or 3 units and therefore he prefers an insured consumption of 2 units.

Another allocation of interest is the one derived from a market for state-contingent claims. Assume that each agent has the utility function $u(s, x) = \log x$ and that the prevailing state is $s^* = a$. Then the equilibrium prices are $(p_a, p_b, p_c) = (1, 16/115, 25/115)$, where p_s is the price of one unit payable in state s, and the resulting allocation of claims is shown on the left in Table III. The construction of this equilibrium depends on the assumption that an agent cannot sell more claims than his endowment and that no agent infers the prevailing state from the prevailing prices. Proceeding symmetrically for each of the other two states that might prevail yields the actual market allocation shown

TABLE III

Agent (i)	State (s):	Allocation of Claims ($s^* = a$) a	b	c	Prevailing state (s^*):	Market Allocation (x115) a	b	c
1		666/115	0	0		666	144	225
2		225/115	0	9		225	666	144
3		144/115	9	0		144	225	666

on the right in Table III. A short computation reveals that this market allocation violates individual rationality. For instance, in the event $\{a, b\}$ agent 3 is worse off with the market allocation than with his endowment. A rational-expectations model would eliminate this difficulty, of course, since each of the two poorly informed agents could infer the state from the prices. In this case there would be no trade at the prices $p = (1, 0, 0)$ when $s^* = a$, and the market allocation would be the endowment. Implicit communication via the market process preserves individual rationality but still it eliminates the kind of favorable insurance arrangement provided by the allocation x in Table II. Notice that the market allocation in Table III could be improved by equalizing the consumptions of the two poorly informed agents in each state.

The allocation in Table II is not immune to criticism. The insurance plan for the two poorly informed agents appears to require the cooperation of the perfectly informed agent regarding states which he knows do not prevail. The possibility of communication raises the prospect that if $s^* = a$ then the coalition of agents 1 and 2 could do better by retaining their endowments, perhaps with agent 2 rewarding agent 1 for saving him the cost of insurance. These matters will be examined further when we study the core in Section 3. I defer the question of what institutionalized process, market or nonmarket, could achieve the allocation in Table II.[3] For now it suffices to observe that the greatest opportunities for insurance are obtained by restricting communication to the null c.s. I conclude, therefore, that a viable definition of efficiency without communication should allow the allocation in Table II to be efficient.

With these examples in mind I turn to a definition of efficiency. I propose that an allocation is efficient iff in each event which every agent can discern there is no other allocation which each agent prefers given his own information. That is, an allocation is efficient iff there is not an event $E \in \bigwedge_N F$ and another allocation $\bar{x}$ such that $\bar{x}_i \succ_{iE} x_i$, namely $\mathscr{E}_i\{u_i[\bar{x}_i]|F_i\} > \mathscr{E}_i\{u_i[x_i]|F_i\}$ on E, for every agent $i \in N$. Note that the null c.s. is imposed. The origin of the requirement that the contingency must be recognized by every agent is evident in Example 2. There we saw that a reallocation of the endowment may extend over states known by some agents not to prevail.

[3] The allocation in Table II can be achieved by a market in state-contingent claims with the fixed prices $p = (1, 1, 1)$ if each agent is prohibited from trading in the claims for which he has perfect, or "inside," information; i.e., agents 1, 2, and 3 are prohibited from trading claims payable in states a, b, and c, respectively.

This notion of efficiency is also called "coarse" efficiency to distinguish it from the weaker concept of "fine" efficiency which admits the full c.s. and allows $E \in \bigvee_N F$. Thus fine efficiency excludes another allocation $\bar{x}$ for which $\mathscr{E}_i\{u_i[\bar{x}_i] \| \bigvee_N F\} > \mathscr{E}_i\{u_i[x_i] \| \bigvee_N F\}$ on $E \in \bigvee_N F$ for every agent $i \in N$. An allocation which is fine *in*efficient on a coarse event $E \in \bigwedge_N F$ is also coarse *in*efficient. In this sense coarse efficiency is a strong requirement. The corresponding notion of strict efficiency is slightly stronger: an allocation x is strictly efficient iff there is not an event $E \in \bigwedge_N F$ and another allocation $\bar{x}$ such that $\mathscr{E}_i\{u_i[\bar{x}_i] \| F_i\} \geqslant \mathscr{E}_i\{u_i[x_i] \| F_i\}$ on E for every agent $i \in N$, with strict preference for at least one agent i on at least one event $A \in PF_i$, $A \subseteq E$. I omit the obvious generalization of the definitions of efficiency to include arbitrary communication systems other than the null and full c.s.

For Example 1 the endowment is both strictly efficient and fine efficient. For Example 2 the allocation in Table II is both strictly efficient and fine efficient. The role of the distinction between coarse and fine events for this allocation will not be apparent until we study the coarse and fine cores in Section 3. This distinction is evident in the market allocation in Table III, however. The market allocation is fine efficient but not coarse efficient; and in fact this is true also of the endowment, which is the market allocation resulting from rational expectations.[4] We see here that fine efficiency is compatible with the "informational efficiency" of market processes (e.g., S. Grossman [**1**] or S. Grossman and J. Stiglitz [**2**]). In contrast, coarse or strict efficiency emphasizes the advantages of insurance, and therefore the disadvantages of direct or implicit communication.

The existence of efficient allocations is easily verified. Consider nonnegative weights $\lambda_i(s)$ for each agent $i \in N$ and each state $s \in S$, and an allocation that maximizes $\Sigma_{i \in N} \mathscr{E}_i\{\lambda_i u_i[x_i]\}$ among the set of feasible allocations. If agent i's weighting function λ_i is F_i-measurable, and not all the weights are zero on any coarse event in $P \bigwedge_N F$, then the allocation is efficient; or if additionally all the weights are positive, strictly efficient. Similarly, a fine-efficient allocation is obtained from $\bigvee_N F$-measurable weights, not all zero on any event in $P \bigvee_N F$. For the coarse-efficient allocation shown in Table II such a set of weights has $\lambda_i(s) = 15$ if s is the state in which agent i has superior information, and $\lambda_i(s) = 6$ otherwise. The extreme form of "ex ante" efficiency which emphasizes insurance to the exclusion of informational considerations is reflected in the allocation shown in Table I for Example 1 and the similar one for Example 2: these allocations correspond to weights which are not only F_i-measurable but in fact constant over the whole set of states.

In most models of market processes the imputed weights are the reciprocals of the agents' marginal utilities of income. Strict efficiency requires, therefore, that each agent's marginal utility of income is measurable with respect to his information. This is just another way of stating the requirement for optimal insurance. In this case the insurance is against what other agents know about the

[4] This is proved in detail in Section 3. See Table V.

prevailing state. A market process vitiates the opportunities for this insurance. In Example 2 the perfectly informed agent is an "insider" in the market for state-contingent claims purchased for insurance purposes by the other two agents, and this distorts the prices and their incomes to the evident advantage of the insider.[5]

Provided the utility functions are differentiable one can state the necessary condition for an interior allocation x to be strictly efficient in terms of the marginal rates of substitution (MRS). Considering only a single commodity and assuming $PF_i'(s)=\{s\}$ for simplicity, agent i's MRS between incomes in states s and $\bar{s}\in PF_i(s)$ is $MRS_i(s,\bar{s})=v_i(\bar{s})/v_i(s)$, where $v_i(s)=u_i'[x_i](s)\mu_i(\{s\})$. Consider a small reallocation such as the cyclic one shown in Table IV for the allocation of Example 2. If x is to be strictly efficient it must be for $(\alpha,\beta,\gamma)>0$ that if $\beta/\gamma\geqslant MRS_1(b,c)$ and $\gamma/\alpha\geqslant MRS_2(c,a)$ so that a marginal reallocation is not unfavorable for agents 1 and 2, then $\alpha/\beta\leqslant MRS_3(a,b)$ so that it is not favorable for agent 3. Allowing negative variations as well yields the necessary condition for strict efficiency that $MRS_1(b,c)\cdot MRS_2(c,a)\cdot MRS_3(a,b)=1$. In general, consider a finite cycle of states $s_1,\ldots,s_K$, $s_{K+1}=s_1$ such that $s_{k+1}\in PF_{i(k)}(s_k)$ for some agent $i(k)$. Then an interior allocation is strictly efficient only if $\Pi_k\, MRS_{i(k)}(s_k,s_{k+1})=1$. This condition is a generalization of the equality of agents' MRS's which is the familiar condition for "ex ante" efficiency in an economy without differences in information.

TABLE IV

Agent (i)	State (s):	a	Reallocation of x b	c
1		5	$2+\beta$	$2-\gamma$
2		$2-\alpha$	5	$2+\gamma$
3		$2+\alpha$	$2-\beta$	5

A substantial part of economic theory is the consequence of the observation that bilateral trade suffices to obtain the equality of the agents' MRS's. Here, it is clear that multilateral trade is necessary, though the institutionalized form that this trading might take is ambiguous. As we saw earlier the missing ingredient of ordinary market processes is some form of "income insurance" which enables each agent i to achieve a marginal utility of income which is F_i-measurable, namely the same for each state $s\in PF_i(s^*)$. There are now a number of well-known examples where the absence of this ingredient has adverse effects; e.g., the study of signalling in labor markets by M. Spence [**1**] and the study of screening in insurance markets by M. Rothschild and J. Stiglitz [**4**]. The substance of the matter is whether institutional arrangements to remedy these effects are possible in principle. In the next section I examine the question by studying the core of an economy with differences in information, and I demonstrate the affirmative answer that the core is not empty.

[5] See footnote 3.

3. THE CORE

In choosing a definition of the core my motive is to identify those allocations having the property that if one is proposed then no subset of the agents has the opportunity and incentive to opt for an alternative allocation. That is, no coalition can block the proposed allocation. When different agents in a coalition have different information their opportunities to take blocking actions jointly are necessarily contingent upon events which they all can discern.

I suggest the following definition of contingent blocking. An allocation is blocked if some coalition can enforce an alternative allocation which they prefer in an event which they all can discern. Specifically, a (nonempty) coalition $M \subseteq N$ can enforce an allocation $\bar{x}$ in an event $E \in \bigwedge_M F$, which its members all can discern, iff $\Sigma_{i\in M}\bar{x}_i(s) = \Sigma_{i\in M} e_i(s)$ for each state $s \in E$; and if $\mathscr{E}_i\{u_i[\bar{x}_i]|F_i\} > \mathscr{E}_i\{u_i[x_i]|F_i\}$ on E for each member $i \in M$ then the proposed allocation x is blocked. The core is then the set of unblocked allocations. Note that this definition confines a blocking coalition to its null c.s.

This can also be called the coarse core to distinguish it from the fine core for which a blocking coalition can also use its full c.s. If each coalition M has a specified set $C(M)$ of feasible communication systems then a general definition can be phrased as follows: an allocation x is blocked iff there is a coalition $M \subseteq N$ having a feasible c.s. $(H_i)_{i\in M} \in C(M)$, and event $E \in \bigwedge_M H$ which its members can all discern using the c.s., and an alternative allocation $\bar{x}$ which it can enforce in the event E and which every member $i \in M$ prefers given the information H_i in the event E, namely $\mathscr{E}_i\{u_i[\bar{x}_i]|H_i\} > \mathscr{E}_i\{u_i[x_i]|H_i\}$ on E for each member $i \in M$.

The definition of blocking invokes three considerations, of which the first is peculiar to an economy with differences in information among the agents. A coalition can block only in an event which every member can discern using some one of its feasible communication systems, since otherwise joint action is not possible. Moreover it can object only with an alternative allocation which it can enforce given that the specified event is known to obtain. And lastly, each member must prefer the alternative allocation based on his information derived from the c.s. in whatever finer event he knows or learns to obtain.

The requirement that a proposed allocation be unblocked is postulated as a minimal desideratum for its stability as a candidate in a negotiating process. One can envision that the agents negotiate the terms of an enforceable contract. Each agent has his private information but in an institutionalized setting he may be unable or unwilling to reveal it. The proposal of an unblocked allocation offers no coalition an opportunity and incentive to object in any contingency. Any other allocation is unlikely to be sustained against counterproposals and ultimately adopted in an event in which a coalition can block; for, each member recognizes the possibility of the event (and its certainty if it is discernable from their null c.s.) and together they have an incentive to opt in favor of their alternative allocation which they can enforce.

For Example 1 the coarse and fine cores consist only of the endowment. For Example 2 the allocation in Table II is in the coarse core. The market allocation in Table III, however, is blocked by agent 3 in the event $\{a, b\}$.

The fine core for Example 2 is actually empty, as I shall now demonstrate. In each state the perfectly informed agent and each two-agent coalition must get at least their endowments since the full c.s. allows them to identify the prevailing state. Thus the endowment is the only candidate for an unblocked allocation in the fine core. But the endowment is blocked by the whole coalition N using its null c.s. in the whole event S by proposing the alternative allocation displayed in Table V, provided that $\varepsilon > 0$ is sufficiently small. Thus the fine core is empty for Example 2. The apparent source of this difficulty is the conflict between the smaller coalitions' use of communication to gain advantages, with the whole coalition's opportunity to provide insurance. It is true in general that the more communication is allowed the smaller is the resulting core, but we see in this example that the tension between the null c.s. and the full c.s. is sufficient to eliminate the core. An analogous conclusion is obtained by M. Rothschild and J. Stiglitz [3] in their study of insurance markets with differential information, where full communication occurs implicitly when an insurer infers a buyer's risk class from the type of contract he purchases.

TABLE V

		Alternative allocation		
Agent (i)	State (s):	a	b	c
1		$5+2\varepsilon$	$2-\varepsilon$	$2-\varepsilon$
2		$2-\varepsilon$	$5+2\varepsilon$	$2-\varepsilon$
3		$2-\varepsilon$	$2-\varepsilon$	$5+2\varepsilon$

It is easy to verify that the coarse core is never empty. The proof is obtained by constructing another cooperative game for which the players are the pairs (i, A) in which $i \in N$ and $A \in PF_i$. A player (i, A) prefers one allocation x to another $\bar{x}$ iff agent i prefers x to $\bar{x}$ given F_i in the event A. The admissible coalitions are those of the form $(M, E) \equiv \{(i, A) | i \in M,\ A \in PF_i,\ A \subseteq E\}$ for $M \subseteq N$ and $E \in \bigwedge_M F$. Such a coalition can enforce the allocation x iff M can enforce it in the event E. It is straightforward to verify that this newly constructed cooperative game is a balanced game as defined by Scarf [5]. Consequently, there exists an unblocked allocation in the ordinary core of this game. Such an unblocked allocation is also unblocked in the economy with differential information. Thus the coarse core is not empty.[6]

The substance of this argument is merely the observation that each agent can wear several hats in the negotiating process; or possibly he can delegate responsibility to subordinates, one for each event in his informational partition, to whom he confers responsibility in that event. This approach will not work, of course, whenever any coalition has access to a non-null c.s. In Example 1, for instance, the economy is usefully regarded as a game among the three players

[6] An alternative proof consists of showing that in the coarse core is an allocation resulting from a constrained market process. For fixed prices $(p(s))$ which equate demands and supplies in each state, each player (i, A) chooses a feasible consumption plan to maximize $\mathscr{E}_i\{u_i[x_i]|F_i\}(A)$ subject to the budget constraint $\Sigma_{s\in A}\, p(s)[x_i(s)-e_i(s)] \leqslant 0$.

$(1,\{a\})$, $(1,\{b\})$, $(2,\{a,b\})$. It is then clear that the endowment is the only allocation in the coarse core, since the first two players will invariably insist on getting their endowments. A similar viewpoint in Example 2 motivates the allocation in Table II, though it is not the only unblocked allocation; and especially, compared to the market allocation in Table III, it motivates the requirement that the outcome of the game be efficient in the coarse sense.

4. AN EXTENSION

A natural objection to the definition of the coarse core is the conjecture that either strategic considerations or the usefulness of information in production might favor the "informational efficiency" of market processes. I conclude briefly, therefore, with a more elaborate construction which addresses the matter to the extent that a version of the coarse core remains nonempty.

Assume that each agent i has a set of feasible decisions which is a compact and convex subset of a finite-dimensional Euclidean space (or more generally, a complete separable metric space). For a coalition M a strategy $d^M=(d_i^M(s))$ specifies for each member $i\in M$ a decision rule d_i^M as a function of the state. Given a strategy d^M the coalition M obtains from production the commodity bundle $y^M(s,d^M(s))=y^M[d^M](s)$ in state s, and each member i has the endowment $e_i(s;d^M(s),d^{-M}(s))=e_i[d^M,d^{-M}](s)$ depending on the strategy of the complementary coalition $-M=N\backslash M$. The coalition can enforce the allocation x in an event $E\in\bigwedge_M F$ iff $\Sigma_{i\in M}x_i[d^M,\ d^{-M}](s)\leqslant\Sigma_{i\in M}e_i[d^M,d^{-M}](s)+y^M[d^M](s)$ for each state $s\in E$ and each strategy d^{-M} of the complementary coalition. Note that the allocation depends upon the strategies of both coalitions.

An outcome is a pair (d^N,x) consisting of a strategy for the whole coalition, such that d_i^N is $\bigvee_N F$-measurable for each agent i, and an allocation that it can enforce in the whole event S. Such an outcome is blocked by a coalition M in an event $E\in\bigwedge_M F$ proposing one of its feasible strategies d^M and an allocation $\bar{x}$ which it can enforce in the event E iff for each member $i\in M$ the decision rule d_i^M is $\bigvee_M F$-measurable and

$$\mathscr{E}_i\{u_i[\bar{x}_i[d^M,d^{-M}]]|F_i\}>\mathscr{E}_i\{u_i[x_i[d^N]]|F_i\}$$

on E for every strategy d^{-M} of the complementary coalition which is $\bigvee_{N\backslash\{i\}}F$-measurable in each component. The core then consists of the unblocked outcomes. (The weak measurability requirement on the complementary coalition's strategy is perhaps unsatisfactory. It envisions that each member of a blocking coalition trusts only that his own information does not leak out to the complementary coalition, since his colleagues may not have motives to withhold.)

Assume that the endowments $e_i[\cdot]$ and the production functions $y^M[\cdot]$ are each continuous and concave, and F_i' and $\bigvee_M F'$-measurable, respectively. Also, if B is a balanced collection of coalitions, namely there exist weights $\alpha_M\geqslant 0$ such that $\Sigma_{i\in M\in B}\alpha_M=1$ for each agent $i\in N$, and $d_i^N=\Sigma_{i\in M\in B}\alpha_M d_i^M$, then

$y^N[d^N] \geqslant \Sigma_{M \in B} \alpha_M y^M[d^M]$. This condition is a consequence of the concavity and homogeneity of y^N if $y^M[d^M] = y^N[d^M, 0]$.

The proof that there exists an unblocked outcome follows the previous argument, supplemented by H. Scarf's [6] construction for cooperative games in normal form.

The following example illustrates some of the features of this formulation.

EXAMPLE 3: Two agents named Row and Column are to play one of two noncooperative games called Left and Right, each equally likely. Only Column knows which game is to be played. Column has no decision to make but Row must choose between two decisions Up and Down. The payoffs (in the single commodity) to Row and Column are shown as ordered pairs in Table VI. The Nash equilibria of this game lead Row to choose Down. This is true also if

TABLE VI

Decision	State: Left	Right
Up	2,2	0,2
Down	0,0	4,0

Column is allowed first to send a message to Row, since Column's incentive is to induce Row to choose Up in either game. In the coarse core of the corresponding cooperative game is the strategy which chooses Up or Down as the state is Left or Right, and which gives to Row an insured payoff of 3 units in either case; indeed Row can let Column make the decision to obtain an insured payoff of 1 unit in either case.

This example illustrates the legal maxim that favors vesting the better-informed agent with the power and consequences of decisions in situations afflicted with moral hazard. The other side of the coin, of course, is the need to insure the poorly informed agent.

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Manuscript received September, 1976; revision received May, 1977.

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