Reputation and Competition with After-sales Services*

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August 2011

Abstract

In this paper we develop a model of reputation and competition. We focus on after-sales services and show that: (i) firms end up with positive profits, thereby being induced to credibly promise high noncontractible quality; (ii) more concentrated industry structures deliver higher quality and increases consumer welfare. The two results are new as they are generated in a pure hidden action model, where homogeneous firms compete à la Bertrand, entry is free and consumers are fully rational in that they infer future quality both from past quality and from current prices.

Keywords: quality, reputation, Bertrand competition, after-sales services.

JEL Codes: L13, L14, D82, D81, C73.

*A previous version of this paper has been circulated under the title "Quality and Reputation: Is Competition Beneficial to Consumers?". We thank Luigi Buzzacchi, Antonio Nicolò, Giancarlo Spagnolo, seminar audience at EARIE 2010 (Istanbul, Turkey), IUSS, Università di Pavia (Italy) and Department of Economics, Università di Brescia (Italy), for useful comments. The usual disclaimer applies. Both authors gratefully aknowledge support from PRIN 2007, (Protocollo 2007R5PN7Y), the second author also from PRIN 2005 (Protocollo 2005137858_001). [a]*Dipartimento di Scienze Economiche, University of Brescia, Italy; e-mail: fedele@eco.unibs.it.* [b]*Istituto di Economia dell'Impresa e del Lavoro, Catholic University of Milan, Italy; e-mail: piero.tedeschi@unicatt.it.*

1 Introduction

Seller reputation consists in buyers' information and beliefs about her/his ability to produce high-quality goods and services. Reputation is a crucial aspect when quality is hard to measure.[1] Indeed, seminal papers on this topic (see, e.g., Klein and Leffler, 1981) argue that high quality, when unverifiable, is not sustainable in one-shot games. By contrast, in on-going relationships clients can react to the choice of providing low quality by not repeating their purchase. Intuitively, this reaction constitutes a punishment for a firm only if providing high quality commands a profit margin, which is called quality premium.

In the current paper we develop a model of reputation and competition and contribute to this literature by proving two main results: (i) we show that competitive firms end up with equilibrium positive profits, which enable to sustain investment in high quality; (ii) we find that more concentrated industry structures can generate a higher consumer welfare. Interestingly, the first result requires a solution to the famous Stiglitz (1989) objection, according to which competition should eliminate all supracompetitive gains, making it difficult for reputational forces to induce the production of high-quality goods. This statement has been a challenge for almost 30 years. Only recently, a few papers provided possible solutions: in the next section we will discuss how our contribution relates to and differentiates from them. The second finding has important policy implications as it should induce cautiousness in advocating that competition has to be enhanced when quality is at stake.

We construct a pure moral hazard setup with homogeneous consumers and firms that interact repeatedly. Unlike most existing literature, we focus on post-sales services instead of physical goods; product quality is determined before than or simultaneously to prices or quantities, while after-sales service quality is determined later, typically in the process of providing the service itself. This is a relevant aspect, which we discuss in the next section, since only the latter timing is fully compatible with a standard moral hazard model where asymmetric information arises after the contract is signed.

The consumers are assumed to be rational in that they infer future noncontractible quality both from past quality and from current prices. We prove the existence of equilibria where positive profits induce firms to credibly promise high quality. This occurs since cheating (i.e. producing a lower than promised quality level) entails the cost of losing market share in the future, due to the existence of an imperfect public signal about firms' quality. The signal can be the result of word of mouth communication or originated by specialized publications or, finally, by forums and discussion groups on the internet.[2]

Our results on quality and welfare are driven by two different forces, depending on the relationship between the equilibrium quality level and the firms' market share. When the former increases with the latter, we consider equilibria which imply the existence of a social convention about the minimum acceptable quality. Such a convention is an equilibrium result and it prevents low quality levels from being accepted.[3] In addition, the incentive compatibility (IC henceforth) constraint requires that

[1] See Bar-Isaac and Tadelis (2008) for a recent survey on reputation literature.

[2] The *Ebay* system of feedbacks, i.e. the ex-post evaluation of sellers (and buyers) made by the counterpart, is an example of the public signal we have in mind. *Trip advisor* is another one.

[3] We use the term of social convention instead of the common one of social norms because the latter has a more

the firms' equilibrium market share is (relatively) large and increasing in the minimum quality level. Finally, the social convention blocks further entry, which is otherwise free in our framework, because potential new entrants would provide quality below the minimum. As a consequence, profits cannot be nought in equilibrium: this explains our first result of positive profits and quality. If, instead, the social convention is not part of the equilibrium or the socially accepted quality is low, then more entry is compatible with the equilibrium: as a consequence quality and firms' profits diminish. In the limit, when quality is not a social issue, each firm has a negligible market share, profits are nil and also consumer welfare is minimum. This argument explains our second result on consumer welfare. Interestingly, also social welfare is negatively affected by competition in this case.

On the contrary, when the equilibrium quality decreases with concentration, then profits are increasing in competition since firms must be compensated for the higher investment in quality. This is shown to negatively affect consumer welfare, hence entry is blocked when the consumers' participation constraint becomes active. The effect on social welfare is instead ambiguous here.

We apply our model to the analysis of a specific industry: insurance. Quality is related to post-sale services: a company can influence utility a customer derives from, say, damage insurance by providing consultancy on various issues, legal, medical, technical, etc., and especially by reimbursing damages promptly. We think that our results are general; yet, we consider the insurance sector because here our focus on (post-sale) services is natural and the quality problem is typically severe. It is indeed easy to show that an insurance company has no incentive to provide positive unverifiable quality to its clients in one-shot relationships. The hypothesis of nonverifiability seems reasonable in the case we deal with: a delay in the reimbursement, for instance, can be justified in a lot of different ways which make it difficult for a Court to understand whether it was done on purpose by a company or it was the result of the natural course of events.

There could be different institutional solutions to this moral hazard problem. For instance, some of the services could be supplied by third parties and not necessarily by the insurer. Delegation could indeed be a good commitment device in presence of quality non-verifiability. Yet, it could also be costly for a company if moral hazard problems of the insured agent and/or the third party, or problems of frauds, are taken into account. This could prevent the insurer from resorting to a delegation option. At the same time, customers often lack the competence to select and hire a suitable third party. What we usually observe, indeed, is that the reimbursement process is managed directly by the insurers, who, together with the policy, sell also legal and technical assistance in the case of damage. These premises reinforce our idea that studying the incentives for an insurance firm to supply a correct quality level of after-sales services is an important economic issue.

The remainder of the paper is organized as follows. Section 2 surveys related literature. Section 3 studies the equilibrium of Bertrand competition among the firms when service quality is verifiable. In Section 4 we relax the verifiability assumption. In Section 5 we analyze the reputational issue. Section 6 discusses the robustness of our results and Section 7 concludes.

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technical and different meaning in the economic literature: see, e.g., Kandori (1992).

2 Related Literature

Our paper has strong connections with the literature on reputation and competition in industrial sectors. As mentioned above, a few recent papers study the effects of competition on quality premium and offer possible solutions to the deriving problem of the erosion of supracompetitive gains. In a mixed model, i.e. a model with both hidden actions and types according to Bar-Isaac and Tadelis (2008) terminology, Hörner (2002) shows that firms end up with positive profits and hence can sustain the production of high quality goods. We differentiate from Hörner (2002) because we explicitly model strategic interaction among firms and, more importantly, we prove the same result of extra-rents in a pure hidden action framework.

However, high quality can be sustained in equilibrium also with zero profits, if the supracompetitive rent originated by good reputation about quality is just sufficient to cover the quality costs: on this respect seminal papers are Klein and Leffler (1981), Shapiro (1983) and Allen (1984). The result of positive equilibrium quality in hidden action models is typically obtained by making specific assumptions about consumers' beliefs. For instance, Shapiro (1983) explicitly states that in his model "consumers' expectations are not fully rational", in that they underestimate quality of new goods, thereby obliging firms to signal high quality through low initial prices. In our framework the focus is on post-sales services: quality is not determined yet when consumers and firms sign the contract. This mitigates the problem of irrational beliefs arising in the Shapiro's setup. In more recent contributions, Tadelis (2002) and Toth (2008) condition quality expectations only to past quality level and not, for instance, to current prices. Again, this helps justify the fact that competition may play no signalling role at the moment of contract signature; this aspect should otherwise be taken into account when quality is decided before the contracting phase.

Kranton (2003) assumes fully rational beliefs, but shows that, absent collusion, a firm can gain by cutting its price, with the effect that quality premia may collapse below the minimum level assuring high quality. Dana and Fong (2010) analyse collusion explicitly and demonstrate that providing lower quality at a lower price would mean to break the cartel agreement, thus starting a price war. This is sufficient to discipline firms in providing high quality. They conclude that oligopoly is the market structure that better sustains high quality, since firms are punished by rivals when lowering price and by consumers when cutting quality. At the opposite, in our paper we prove that reputation can sustain high quality also when we assume fully rational expectations (differently from Shapiro, Tadelis and Toth) on part of consumers and without resorting to firms' collusion (differently from Dana and Fong). In our model the mere repetition of the game is sufficient to get positive quality. Bar-Isaac (2005) finds an opposite result with respect to Dana and Fong's: he demonstrates that high quality can be sustained in markets where competition is either low or high, but not where it is intermediate. He measures competition through a degree of substitutability, thereby requiring both vertically and horizontally differentiated products.[4] Our approach differs in that we disregard horizontal differentiation and we find a monotonically positive relationship between quality and

[4] An analogous assumption is made by Vanin (2009a; 2009b), who introduces a linear demand oligopoly model with free entry and finds that high quality is provided only when it is sufficiently important to buyers.

concentration.

Toth (2008) focuses on the evolution of market structure by modelling a firm's investment in quality as a mean of increasing its probability of survival. When market is competitive the importance of survival is low, with the effect that the firms invest less in quality: this speeds up the convergence to a more concentrated market structure. Oligopolistic firms have then a bigger incentive to invest in quality. Yet, free entry reduces concentration and may prevent firms from investing in high quality. In common with us, Toth (2008) shows that quality is increasing with concentration.[5] However, he considers less sophisticated consumers, as mentioned above; moreover, our proofs work through different mechanisms.

Our paper has also connection with the literature on the administrative costs of insurance companies. Gollier (1992, 2000) indicate marketing costs, management costs and costs to audit claims as main sources of administrative expenses for insurers. Such costs are generally modelled as a function of the contractual level of reimbursement. By contrast, we model the quality level as directly influencing them. Our different approach is justified by the fact that quality of after-sales insurance services is closely connected with hiring experts in order to verify damages, having better call centres and/or agencies, having a policy for quick indemnity, better attorneys to assist the clients, etc.: all these aspects generate costs which increase with the service quality but, as a first approximation, do not depend on the level of reimbursement. Other papers study how administrative costs drive the optimal design of insurance contracts. Raviv (1979) and Blazenko (1985), for example, show the optimality of coinsurance above a deductible when administrative costs are a strictly convex function of the reimbursement. Their contributions help formulate the model, but they have no bearing on the interpretation of our results.

Finally, literature on self-insurance is relevant for our paper. In a seminal work, Ehrlich and Becker (1972) define self-insurance as investments made by clients in order to reduce the severity of damages. The problem we address becomes very similar to that of self-insurance if we interpret the lack of quality as an additional damage borne by the consumers. We could then allow the clients to buy insurance for the monetary damage and to side-contract insurance on the additional "quality" damage. However, this interpretation does not add insights to our analysis, as argued in Section 3.

3 Competition with Verifiable Quality

Consider an economy with a continuum of risk-averse consumers (also referred to as clients or customers) and $n \geq 2$ insurance companies (also referred to as insurers or firms). Each consumer is characterized by the following expected utility function:

$$\tilde{U} \equiv pU(W - D + R, S) + (1 - p) U(W - \beta R), \tag{1}$$

where: $p \in (0, 1)$ is the probability the consumer suffers a damage; W is the consumer's initial wealth; D is the monetary value of the damage; R is the contractual level of a reimbursement provided by

[5] See also Calzolari and Spagnolo (2010) for a similar result in a different setup.

an insurance company; S indicates the monetary equivalent of an after-sales service, provided by an insurer, which is able to reduce material and psychological costs borne by the consumer on top of the damage D (a quick payment of the amount R is an example); βR is the premium and therefore β is the premium ratio. The role of S is to measure the quality of service: the lower S, the worse the service quality; $S = 0$ denotes a minimum legal standard, or a level below which under-provision of quality can be easily verified by a Court. Finally, U is a vN-M utility function with $U_W > 0 > U_{WW}$, $U_S > 0 > U_{SS}$, $U_{WS} < 0$ (subscripts W and S denote partial derivatives); $U(x)$ is a (slightly abusive) shortcut for $U(x, 0)$. Signs of first and second order derivatives are standard; sign of the mixed one is a sufficient condition for the demand for reimbursement R to decrease with the quality level S: this a form of substitutability between quality and wealth.

Each insurer is characterized by the following expected profit function:

$$\tilde{\Pi} \equiv [(1-p)\beta - p] R - pc(S), \tag{2}$$

where $c(S)$ denotes the administrative costs of the provision of a service with quality S, with $c(\cdot)$ twice differentiable, $c' > 0$, $c'' > 0$ and $c'(0) \to 0$.

In this section we deal with the benchmark situation where service quality S is verifiable, therefore also contractible. We define the first best contract as the solution to the following problem: a representative consumer maximizes his expected utility $\tilde{U}$ subject to the participation constraint of a representative insurer, whose outside option is assumed to have zero value, i.e. $\tilde{\Pi} \geq 0$.[6] We also show that the equilibrium of a one-shot Bertrand competition game among the insurers replicates the first best.

The timing of the Bertrand game is as follows:

- insurers compete à la Bertrand, making simultaneous offers of R, β and S;
- each consumer either selects the preferred triple or buys no insurance;
- Nature selects the state of the world for each consumer (i.e. damage or no damage);
- the accepted contracts are implemented.

Lemma 1 *The Bertrand equilibrium contract when S is verifiable has the following features:*

1. *the insurers get zero expected profits;*
2. *the level of service quality is positive;*
3. *the consumers obtain maximum expected utility.*
4. *The equilibrium contract is characterized by the following conditions:*

$$\begin{cases} c'\left(S^{FB}\right) = \frac{U_S\left(W-D+R^{FB}, S^{FB}\right)}{U'\left(W-\beta R^{FB}\right)}, \\ U_W\left(W-D+R^{FB}, S^{FB}\right) = U'\left(W-\beta R^{FB}\right), \\ (1-p)\beta^{FB} = p\frac{R^{FB}+c\left(S\left(\beta^{FB}R^{FB}\right)\right)}{R^{FB}}, \end{cases} \tag{3}$$

[6] Throughout the paper we refer to each consumer as "he" and to each insurer as "she".

where superscript FB stands for first best.

Proof. See appendix A.1. ■

In order to interpret Lemma 1, we resort to the usual notions of full insurance and actuarially fair premium, but focusing on the case where service quality is not an issue, i.e. when S is not an argument of the consumers' utility function. Full insurance is then defined as the amount of R that equates consumers' monetary wealth across states:

$$W - D + R = W - \beta R \Leftrightarrow R = \frac{D}{1+\beta}.$$

In our model, the fair rate is $\beta = \frac{p}{1-p}$ and induces the consumers to buy full insurance, as one can check by substituting it into the first order conditions. The implication of Lemma 1 can now be readily seen as standard. Indeed

$$U_W\left(W - D + R^{FB}, S^{FB}\right) = U'\left(W - \beta R^{FB}\right)$$

is the usual condition of optimal insurance, i.e. the marginal utility of wealth must be equal across states. This implies that if $U_{WS}(W - D + R, S)$ were nought, the optimal contract would entail full insurance, i.e. $R = \frac{D}{1+\beta}$. By contrast, since quality and wealth are substitutes, we observe partial insurance, i.e. R^{FB} is lower than $\frac{D}{1+\beta}$, given that the premium is unfair, i.e. $\beta^{FB} > \frac{p}{1-p}$. The result on the premium is typical of optimal insurance contracts when there are administrative costs of any type. Indeed, a fair premium would induce negative profits, thereby violating the firm's participation constraint.[7]

4 Competition with Unverifiable Quality

In this section we relax the assumption of verifiability of the service quality. As argued in the introduction, services are supplied by the insurance companies after the contracts are signed and the consumers are damaged: quality S can be observed by consumers, but it is reasonable to assume that it is not verifiable by third parties. This means that contracts cannot be conditioned on S. As a consequence, in one-shot relationships the companies have an incentive to renege a promised positive quality level after signing a contract (R, β). Indeed, as $\tilde{\Pi} \equiv [(1-p)\beta - p] R - pc(S)$ is decreasing in S for any couple (R, β), the minimum possible level $S = 0$ will be the profit-maximizing choice.

Taking into account the above reasoning, we replicate the analysis of Section 3, by studying a one-shot Bertrand competition game among the insurers, under the new hypothesis of nonverifiability of S. The time structure of the game is as in Section 3.

[7] The model presented in this section can be interpreted in another way. Since wealth and service quality are substitutes, it is natural to interpret the lack of S as an additional damage borne by the consumers. Therefore the consumers pay a unique premium and obtain two different services. One could ask whether adding another price (the price of the service) would affect efficiency of the contract. Yet, it can be proved that there is no gain in doing so. It follows that the first best contract of this section is the correct reference point for all the subsequent analysis. This alternative way to present the model highlights its similarity to that of self insurance, where consumers insure specific risks without resorting to an insurance company; adding the price of the service in our model would depict a situation where consumers buy side insurance from the same company.

Lemma 2 *The Bertrand equilibrium contract when S is not verifiable has the following features:*

1. *the insurers get zero expected profits;*
2. *the level of service quality is nought;*
3. *the consumers obtain the maximum expected utility subject to $S = 0$.*
4. *The equilibrium contract is characterized by the following equations:*

$$\begin{cases} S^{\circ} = 0, \\ R^{\circ} = (1-p)\,D, \\ (1-p)\,\beta^{\circ} = p. \end{cases} \tag{4}$$

Proof. See Appendix A.2. ■

By comparing contract (4) to contract (3), characterized in Lemma 1, we can verify that (i) the firms' participation constraints are satisfied with equality under both arrangements, i.e. their profits are nought; (ii) contract (4) entails full insurance and fair premium; (iii) the clients' expected utility is lower under the former because S is zero, whereas the utility is maximized at S^{FB}, which is generically strictly higher than zero. We can conclude that the equilibrium contract when S is not verifiable entails unexploited gains from trade.

5 Contracting and Competition with Reputation

In this section we investigate under which conditions reputational concerns may induce the firms to provide a positive level of (nonverifiable) quality and may reduce the magnitude of unexploited gains from trade discussed at the end of Section 4. We first study the contracting problem between a representative insurer and her customers, focusing on a setting where the latter are given full bargaining power. This is done in order to provide an appropriate benchmark for the subsequent analysis of a Bertrand competition game among the insurers. Then we extend the analysis to competition.

5.1 Contracting

We consider a repeated interaction among infinitely lived clients and a representative insurer.[8] This requires a few extra-assumptions. First, clients have to renew the insurance also in the event the damage D does realize (one can think of a public liability insurance). Furthermore, we let 1 be the measure of clients and $\sigma_{i,t}$ the fraction of consumers served by insurer $i = 1, ..., n$ at time $t = 0, ..., \infty$, i.e. her market share, so that $p\sigma_{i,t}$ is the fraction of damaged clients of insurer i at time t. In each period $t \geq 0$, contracting between insurer i and her customers takes place according to the following timing:

- a representative consumer offers a contract $\{R_{i,t}, \beta_{i,t}, S_{i,t}\}$ to his insurance company;

[8] Nothing substantial would change if the assumption of infinitely lived clients were relaxed, provided that in any period the measure of those entering the market is equal to that exiting it.

- insurer i either accepts the contract or refuses it;

- Nature selects the state of the world for each customer;

- insurer i selects a level of $S_{i,t}^{A}$ for each damaged customer, where superscript A stands for actual;

- in each period Nature selects the following public signal: with probability $\varphi(\tau_{i,t}, \sigma_{i,t}) \in [0,1]$ *all* consumers receive a signal of bad quality, where $\tau_{i,t}$ is the share of clients who enjoy an after-damage quality level $S_{i,t}^{A}$ lower than the contracted $S_{i,t}$, i.e. the cheated clients;

- if customers receive a signal of bad quality, they know that insurer i cheated somebody; they then decide whether to renew the contract or not.

The above timing depicts a moral hazard model, where the hidden effort is the actual level of quality provided by the insurers after the contract is signed. We put restrictions on the functional form of the public signal probability $\varphi(\tau_{i,t}, \sigma_{i,t})$.

Condition 1 *(i)* $\varphi(0, \sigma_{i,t}) = 0$*: if insurer i cheats no clients, no signal is conveyed, i.e. we rule out the possibility that a non-cheated customer sends a signal of bad quality;*[9]

(ii) $\varphi_{\tau}(\tau_{i,t}, \sigma_{i,t}) > 0$*, with* $\varphi_{\tau}(0,0) = 0$*, and* $\varphi_{\tau\tau}(\tau_{i,t}, \sigma_{i,t}) \geq 0$*, where subscript* τ *(and* σ *below) denotes partial derivative: probability* φ *is increasing and nonconcave in the fraction* τ *of cheated clients;*

(iii) $\varphi_{\tau\sigma}(\tau_{i,t}, \sigma_{i,t}) > 0$*: this condition has a simple interpretation which will be discussed below.*[10]

At time t the discounted value of insurer i's profit is

$$V_{i,t} \equiv \sigma_{i,t} \left[\tilde{\Pi}_{i,t} + p\tau_{i,t} \left(c(S_{i,t}) - c\left(S_{i,t}^{A}\right) \right) \right] + \delta \left[1 - \varphi(\tau_{i,t}, \sigma_{i,t}) \right] V_{i,t+1}, \tag{5}$$

where δ is the discount factor. A trade-off is faced now by the firms: when cheating $p\tau_{i,t}\sigma_{i,t}$ consumers at any time $t \geq 0$, insurer i saves the amount $p\tau_{i,t}\sigma_{i,t} \left(c(S_{i,t}) - c\left(S_{i,t}^{A}\right) \right)$ on administrative costs, but she incurs the expected loss $\delta\varphi(\tau_{i,t}, \sigma_{i,t}) V_{i,t+1}$ of future profits if no client renews the contract when receiving the signal of bad quality. We will see that this is the consumers' equilibrium behavior: turning to another insurer is indeed the optimal strategy for them since they believe, upon receiving the signal, that the company will cheat with probability 1.

In this context, Lemma 3 computes the parametric conditions for which the insurers find it profitable not to cheat any customer, i.e. $\tau_{i,t} = 0$ for any insurer i at any time t. Such a condition defines the firms' IC constraint.

[9] Ebay has recently changed the feedback rules because of evidence that feedbacks were used to threaten the counterpart with the aim of obtaining better contractual conditions. In our paper this is supposed not to occur for either forums are able to check peer information before posting it, or the public signal is sent by specialized publications/sites with a valuable reputation to defend.

[10] Notice that all the properties required for the public signal probability are satisfied by the following simple explicit formulation: $\varphi(\tau_{i,t}, \sigma_{i,t}) = \tau_{i,t} \cdot \sigma_{i,t}$.

Lemma 3 *No insurer decides to cheat her customers if and only if her market share is relatively high, i.e. iff* $\sigma \geq \underline{\sigma}$*, where* $\underline{\sigma}$ *is the market share that satisfies with equality the following condition:*

$$\varphi_{\tau}(0,\sigma) \geq \frac{1-\delta}{\delta}\frac{pc(S)}{\tilde{\Pi}}. \tag{6}$$

Proof. See appendix A.3. ■

Note that the LHS of (6) is increasing in σ since $\varphi_{\tau\sigma}(0,\sigma) > 0$, whilst the RHS does not depend on it: the result of Lemma 3 relies thus on inequality *(iii)* of Condition 1, which can be interpreted as follows. Consider an insurer who decides to cheat additional customers, i.e. to raise τ: the probability that all her customers receive the signal (in which case no consumers will renew the contract) increases since $\varphi_{\tau}(\tau,\sigma) > 0$. Such a variation, in turn, rises with the market share because $\varphi_{\tau\sigma}(\tau,\sigma) > 0$. If market share is relatively high (i.e. $\sigma \geq \underline{\sigma}$), the insurer finds it unprofitable to cheat any customer in order to save on administrative costs for such a positive effect on $V_{i,t}$ is outdone by the negative one due to a large probability that no consumer will renew the contract.

Before proceeding with the analysis it is useful to introduce the following

Remark 1 *The IC condition* (6) *has other implications besides the one on the market share* σ*, according to which the contract is not incentive compatible if* σ *is smaller than* $\underline{\sigma}$*, given* β*,* R *and* S*. Indeed, recalling that* $\tilde{\Pi} \equiv [(1-p)\beta - p]R - pc(S)$*, the RHS of* (6) *increases with* R *and* S *and decreases with* β *ceteris paribus, while the LHS does not depend on any of these three variables. As a consequence, the contract is not incentive compatible also when better contractual conditions are required by a consumer, given his firm's market share* σ*. The consumer, anticipating this, is able to formulate expectations on how the insurance company will behave in the event of a damage. Finally, the following way of rewriting the IC condition*

$$\tilde{\Pi} \geq \frac{1-\delta}{\delta}\frac{pc(S)}{\varphi_{\tau}(0,\sigma)} \tag{7}$$

highlights its similarity to the idea of quality premium (Klein and Leffler, 1981). Indeed, condition (7) *states that the insurers must receive a positive profit on each contract in order to behave (the RHS of* (7) *is strictly higher than zero if* $S > 0$ *and* $\delta < 1$*): if profits were nought the fear of foregoing future profits would not discipline firms to deliver high quality services.*

As one can see by inspecting (5) the choice of $(\beta_t, R_t, S_t, \sigma_t)$ influences only V_t and does not have any impact on V_{t+1}: this means that the model is stationary. The second best contract is hence defined as the solution to the following problem. A representative client maximizes his single-period expected utility $\tilde{U}$, subject to a representative firm's participation constraint plus IC constraint characterized in Lemma 3. We show that concentration benefits the consumers. In the limit, the optimal market structure is monopoly. In this case it would not be reasonable to allocate all the bargaining power to the consumer. Yet, as already anticipated, here we are simply building up a benchmark case, necessary to develop the subsequent analysis of competition.

Lemma 4 *The optimal contract with reputation when quality S is observable but not verifiable is characterized by:*

$$\begin{cases} c'(S^r) = \frac{\delta\varphi_\tau(0,\sigma)}{1-\delta+\delta\varphi_\tau(0,\sigma)}\frac{U_S(W-D+R^r,S^r)}{U'(W-\beta^r R^r)}, \\ U_W(W-D+R^r,S^r) = U'(W-\beta^r R^r), \\ (1-p)\beta^r = p + p\frac{1-\delta+\delta\varphi_\tau(0,\sigma)}{\delta\varphi_\tau(0,\sigma)}\frac{c(S^r)}{R^r}, \end{cases} \tag{8}$$

where superscript r stands for reputation. Moreover, the optimal market structure is monopoly, i.e.

$\sigma = 1$.

Proof. See appendix A.4. ■

Before commenting on the results of Lemma 4, we stress that they can be almost replicated as outcome of Bertrand competition, the analysis of which is in Subsection 5.2. The only difference is that at least two insurers are needed in order to preserve competition: this amounts to require $\sigma_i \leq \frac{1}{2}$ when firms are symmetric.

Contract (8) satisfies with equality the IC constraint. This implies, taking condition (7) with equality, that the firms' single-period profit on each contract is positive and equal to

$$\tilde{\Pi} = \frac{1-\delta}{\delta}\frac{pc(S)}{\varphi_\tau(0,\sigma)}.$$

Since firms' profits are positive and social welfare, defined as the sum of profit $\tilde{\Pi}$ of a representative insurer and utility $\tilde{U}$ of a representative consumer on a single contract, cannot be bigger with respect to the first best scenario, the consumers' utility is lower under contract (8) than under contract (3). This is due to the nonverifiability of the service quality, which imposes a positive lower bound on the level of firms' profits in order for them to behave: the consumers are simply paying the cost of moving from the first to the second best. However, contract (8) represents a Pareto improvement with respect to that without verifiability, i.e. system (4). Indeed, the latter satisfies the IC constraint at $S = 0$. Yet, the consumers do not select it when reputation can be built up: this means that they are better off when choosing contract (8), but also the insurance companies are (since they make positive profits instead of zero ones). As a consequence, contract (8) dominates contract (4) and it is able to reduce the magnitude of unexploited gains from trade discussed at the end of Section 4.

In addition, the IC constraint enables us to explain why monopoly arises as the optimal market structure. Indeed, the LHS of (6) is maximum for $\sigma = 1$. On the contrary, the RHS increases with R and S and decreases with β, ceteris paribus. It follows that as σ augments the insurer finds it profitable not to cheat even for bigger values of R and S or smaller values of β, ceteris paribus. In other words, the higher an insurer's market share σ, the less binding the IC constraint (6) and, in turn, the higher the consumers' expected utility.

Before proceeding with the analysis of competition, we study how the quality level S is affected by the market share σ at the optimum described by (8): this will be crucial to derive some of the subsequent results.

Lemma 5 *A necessary and sufficient condition for the quality level S^r to increase with the market share σ is*

$$-U_{WS} < \frac{c'}{c} U_W \left(\frac{1-p}{p} \frac{U_{WW}}{U''} + 1 \right) - \frac{U_S}{U_W} U_{WW}. \tag{9}$$

By contrast, S^r decreases with the market share σ if and only if

$$-U_{WS} > \frac{c'}{c} U_W \left(\frac{1-p}{p} \frac{U_{WW}}{U''} + 1 \right) - \frac{U_S}{U_W} U_{WW}. \tag{10}$$

Proof. See appendix A.5. ■

In order to interpret the above result, we have first to clarify the meaning of the two conditions. To fix ideas, suppose that $\frac{c'}{c} \to 0$: this condition depicts a situation where variable quality costs are negligible with respect to total quality costs. In such a case (9) and (10) rewrite as

$$- \left. \frac{\partial W}{\partial S} \right|_{U=const} = \frac{U_S}{U_W} > \frac{U_{WS}}{U_{WW}} = - \left. \frac{\partial W}{\partial S} \right|_{U_W=const}, \tag{11}$$

$$- \left. \frac{\partial W}{\partial S} \right|_{U=const} = \frac{U_S}{U_W} < \frac{U_{WS}}{U_{WW}} = - \left. \frac{\partial W}{\partial S} \right|_{U_W=const}, \tag{12}$$

respectively. The LHS of each inequality is the slope (in absolute value) of the indifference curve in plane (S, W), while the RHS is the slope (in absolute value) of the locus of points for which the marginal utility of wealth is constant. Clients' allocations in the good and in the bad states are on the latter curve, since the optimal contract (S^r, R^r, β^r) implies constant marginal utility of wealth. It can be easily shown that if the locus of points with constant marginal utility is flatter than the indifference curve, this is what inequality (11) states, then the clients' utility in the bad state is higher than that in the good one. This amounts to say that at the optimal contract (S^r, R^r, β^r) clients value good services more than monetary transfers. By contrast, condition (12) requires that the indifference curve is flatter, in which case the clients' utility in the bad state is lower than that in the good one, i.e. monetary transfers are more important to consumers than after-sales services.

We are now able to interpret the results of Lemma 5. Condition (11) is sufficient to have S^r increasing with σ. When high quality services are relatively valuable to consumers, then only firms with high market share can profitably and credibly transfer utility through quality instead of money: indeed, the IC constraint (6) holds only for relatively high σ. The result is a fortiori true if $\frac{c'}{c} \geq \varepsilon > 0$, i.e. if variable quality costs are nonnegligible.

On the contrary, condition (12) is necessary and sufficient for S^r to decrease with σ if variable quality costs are small ($\frac{c'}{c} \to 0$). This result may appear surprising given that bigger firms have less incentive to cheat, ceteris paribus, according to IC constraint (6). Yet, the moral hazard problem, and consequently the IC constraint, are less important in this context. Indeed, on the one hand clients are not very concerned about quality; on the other hand, quality costs are almost fixed, hence promising high quality is easily credible.

5.2 Competition

In this subsection we introduce Bertrand competition among the insurers to study the characteristics of the equilibrium contract(s). The following infinitely repeated game with free entry is played by firms and consumers:

a. before $t = 0$ the insurance companies decide whether to enter the market;

b. In each period $t \geq 0$ the insurance companies compete à la Bertrand on R_t, β_t, τ_t, σ_t and S_t: the level of quality S_t is observable but not verifiable, contrary to all of the other variables, hence it is just promised by the insurers;

c. in each period the clients either select the preferred contract or buy no insurance;

d. in each period Nature selects the state of the world for each customer;

e. in each period the insurance companies select a level of S_t for each damaged customer;

f. in each period Nature selects the public signal;

g. then the game starts again from stage b.

We solve the game by focusing on symmetric Subgame Perfect Equilibria (SPEs, henceforth) in pure strategies. Symmetry means that all the insurers choose the same market share σ. This implies that $\sigma = \frac{1}{n}$, with $n \geq 2$. Given this time structure, we can state the following

Claim 1 *A SPE of the game described above, where the insurers compete repeatedly à la Bertrand, S is not verifiable and reputational concerns are taken into account, displays the following features:*

(i) the equilibrium contract is as in (8)*, with the only difference that the market share is not* 1*; rather it is lower than* 50%*, i.e.* $\sigma_i \leq \frac{1}{2}$ *for any* i*;*

(ii) the insurers get positive expected profits;

(iv) the service quality level is positive;

(v) provided that $n \geq 2$*, any increase in the number of firms would result in a reduction of the consumers' utility.*

The precise description and the proof of the equilibrium strategies is postponed to subsequent Proposition 1. Here we wish to comment the qualitative features of the equilibrium contract. First of all, like in Hörner (2002), firms end up with positive profits, even with Bertrand competition and free entry. Furthermore, we show that tougher rivalry among firms in terms of number of competitors in the market leads to a decreased consumer welfare. The reason for this result is as follows. The equilibrium contract is such that the IC constraint is binding. Therefore, as remarked in the discussion of Lemma 4, the softer the competition, i.e. the higher σ, the higher the LHS of (6); this implies in turn that less competitive firms offer better contractual conditions to the consumers, while maintaining a credible commitment to behave.

We now state formally the equilibrium strategies which sustain the equilibrium.

Proposition 1 *There exists a SPE of the infinitely repeated game described above displaying the following features:*

1. *if* $\frac{\partial S^r}{\partial \sigma} > 0$ *then* $\hat{n}$ *(which must be weakly higher than* 2*) denotes the equilibrium number of insurers and it is defined as*

$$\hat{n} = \max\left\{n \in N : \hat{S}(n) \geq \bar{S} > 0\right\} : \tag{13}$$

N is the set of natural numbers, $\hat{S}(n)$ *is the equilibrium (promised) quality level, determined implicitly in the subsequent point 2,* $\bar{S}$ *is a threshold quality level whose meaning and role will be specified below;*

by contrast, if $\frac{\partial S^r}{\partial \sigma} < 0$ *then* $\breve{n}$ *(*≥ 2*) denotes the equilibrium number of insurers and it is defined as*

$$\breve{n} = \max\left\{n \in N : \tilde{U}(n) \geq pU(W-D) + (1-p)U(W)\right\} :$$

$\tilde{U}(n)$ *is the consumers equilibrium utility, determined implicitly in the subsequent point 2,* $pU(W-D) + (1-p)U(W)$ *is the consumers utility when no insurance is bought;*

2. *on the equilibrium path, all insurance companies offer a contract characterized by:*

$$\begin{cases} c'\left(\hat{S}(n_k)\right) = \frac{\delta\varphi_\tau\left(0,\frac{1}{n_k}\right)}{1-\delta+\delta\varphi_\tau(0,n_k)} \frac{U_S\left(W-D+\hat{R}(n_k),\hat{S}(n_k)\right)}{U'\left(W-\hat{\beta}(n_k)\hat{R}(n_k)\right)}, \\ U_W\left(W-D+\hat{R}(n_k), \hat{S}(n_k)\right) = U'\left(W-\hat{\beta}(n_k)\hat{R}(n_k)\right), \\ (1-p)\hat{\beta}(n_k) = p + p\frac{1-\delta+\delta\varphi_\tau(0,n_k)}{\delta\varphi_\tau\left(0,\frac{1}{n_k}\right)} \frac{c\left(\hat{S}(n_k)\right)}{\hat{R}(n_k)}, \end{cases} \tag{14}$$

where n_k *equals* $\hat{n}$ *if* $\frac{\partial S^r}{\partial \sigma} > 0$ *and* $\breve{n}$ *if* $\frac{\partial S^r}{\partial \sigma} < 0$*;*

3. *out of equilibrium path, that is if* $n > n_k$*, all insurers offer the contract of Lemma 2:*

$$\begin{cases} S^\circ = 0, \\ R^\circ = (1-p)D, \\ (1-p)\beta^\circ = p; \end{cases} \tag{15}$$

4. *consumers accept contract* (14) *if* $n \leq n_k$ *and accept contract* (15) *otherwise. Moreover, they refuse any other contract.*

Proof. See appendix A.6. ■

The equilibrium contract (14) is equivalent to contract (8) with the only exception that the market share σ is $\frac{1}{n_k}$ in the former and 1 in the latter. Moreover, it is characterized by a positive level of quality and an upper bound to the number of insurers. These two results are driven by different forces, depending on the sign of derivative $\frac{\partial S^r}{\partial \sigma}$, hence we treat them separately.

(i) If $\frac{\partial S^r}{\partial \sigma} > 0$, then $n_k = \hat{n}$ and the equilibrium contract (14) relies upon implicit expectations, an example of which is given by the following expression:

$$\Pr(i \text{ cheats } | \beta_i, R_i, S_i, \sigma_i) = \begin{cases} 0 & \text{if} \quad \varphi_\tau(0,\sigma_i) \geq \frac{1-\delta}{\delta} \frac{pc(S_i)}{\tilde{\Pi}(\beta_i,R_i,S_i)} \text{ and } S_i \geq \bar{S} \\ 1 & \text{otherwise;} \end{cases} \tag{16}$$

According to (16), the consumers anticipate that a company will not cheat them ($\Pr(\cdot) = 0$) only if the contract proposed by any insurer i, (R_i, β_i, S_i), (i) satisfies her IC constraint for any given σ_i and (ii) provides a level of quality weakly higher than $\bar{S}$.[11] Level $\bar{S}$ can be interpreted as a socially accepted quality standard, which becomes effective when the firms compete repeatedly in an infinite time horizon. If the IC constraint is instead not satisfied and/or an insurer tries to offer a quality lower than $\bar{S}$, the consumers believe that she wants to cheat all of them, by choosing an even worse quality level setting. The reason for this is obvious when the IC constraint is violated, whilst an explanation is needed if quality level is lower than $\bar{S}$. According to the equilibrium strategy, the consumers, taking into account that any other competitor is fulfilling the socially accepted quality standard by offering (14), expect that none of the current clients will accept a contract with quality less than $\bar{S}$: the insurer's market share will lower significantly, with the effect that her IC constraint will be not satisfied. As a consequence, each individual consumer will expect the minimum quality level and will not accept the contract. This lower bound on quality along with the hypothesis that S^r decreases with the number n of insurers active in the market, drive the result on the equilibrium number of firms, $\hat{n}$.

(ii) By contrast, if quality is negatively affected by market share σ, $\frac{\partial S^r}{\partial \sigma} < 0$, the equilibrium number of insurers is $n_k = \breve{n}$ and contract (14) can be explained directly referring to the IC constraint (6), taken with the equality sign:

$$(1 - \delta)\, pc\,(S^r) = \delta \varphi_\tau\,(0, \sigma)\, \tilde{\Pi}. \tag{17}$$

Free entry reduces σ, in which case the RHS of (17) decreases ceteris paribus, since $\varphi_{\tau\sigma} > 0$, while the LHS increases since $\frac{\partial S^r}{\partial \sigma} < 0$. As a consequence $\tilde{\Pi}$ must increase in order to meet equality (17). Since profits augment even after taking into account the increased quality costs faced by more competitive firms, consumers' utility must necessarily diminish. Entry will then be blocked when the required level of $\tilde{\Pi}$ makes active the consumers' participation constraint.

An interesting result can be derived from Proposition 1. Recall that contract (14) satisfies with equality the IC constraint (6) when market share σ is equal to $\frac{1}{n_k}$. This implies that very competitive market structures entail low consumer welfare. At the equilibrium with $\breve{n}$ firms in the market (i.e. when $\frac{\partial S^r}{\partial \sigma} < 0$), such a result derives straightforwardly from the above reasoning. In this case firms profits are instead increasing with competition, hence the effect on social welfare is ambiguous. On the contrary, at the equilibrium with $\hat{n}$ firms in the market (i.e. when $\frac{\partial S^r}{\partial \sigma} > 0$), if the lower bound $\bar{S}$ on quality decreases, the equilibrium number of firms increases according to (13). Therefore, the IC constraint implies that, ceteris paribus, firms find it profitable not to cheat only for smaller values of R and S or bigger values of β: this makes consumers worse-off. At the same time, the RHS of (7) is nought for $S = 0$, hence equilibrium profit $\tilde{\Pi}$ approximates zero as $\bar{S}$ tends to zero. We can correctly conclude that at this equilibrium, also social welfare is negatively affected by competition.

We believe that also other kinds of competition might deliver our results, even though somehow weakened. Focus, for instance, on the equilibrium with implicit expectations (16), according to which

[11] Our argument would work even if, in the second line of (16), we assumed a strictly positive probability, instead of 1, of being cheated: setting it to 1 simplifies the intuition.

quality and welfare decrease with competition. Since insurance premia are bound to the minimum level satisfying the IC constraint (given our Bertrand competitive environment), only positive effects (higher quality) arise of a concentrated market structure, while bad ones (higher prices) are absent or negligible. Nonetheless, the equilibrium quality is very low when many firms are active in the market and this might outdo the beneficial effects of lower prices (high production levels) arising, e.g., with Cournot competition. In addition, we think that price competition is more suitable to describe strategic interaction in the service sector.

6 Discussion of the Results

We believe we owe a discussion of two separated but interconnected issues that could be problematic to our analysis: uniqueness and robustness of the equilibria.

Uniqueness. The equilibrium with $\hat{n}$ firms in the market, which arises when $\frac{\partial S^r}{\partial \sigma} > 0$, depends on the implicit expectations (16). There obviously are other consumers' expectations which sustains different equilibria. Even limiting the analysis to the proposed expectations, different equilibria are sustained, depending on the value of social standard for quality, which is not unique in our equilibrium. With lower $\bar{S}$, that is when clients expect to receive lower levels of quality, we would observe a bigger number of firms in equilibrium, lower industry profits and lower quality levels. On the contrary, the equilibrium with $\breve{n}$ firms in the market, which relies on condition $\frac{\partial S^r}{\partial \sigma} > 0$, is unique as it is determined by the consumers' participation constraint.

Robustness. Our results depends crucially on property (iii) of Condition 1, $\varphi_{\tau\sigma} > 0$. We therefore discuss how our findings are affected when relaxing such an assumption.

Suppose $\varphi_{\tau\sigma}$ is negative. The IC constraint (6) must therefore be rewritten as $\sigma \leq \underline{\sigma}$, with the effect that it would not be violated by a reduction of σ due to entry of new firms. This means that the optimal contract entails $\sigma = 0$, i.e. "extreme" competition. As a consequence, a new equilibrium arises where quality level is nil. Indeed, by substituting $\sigma = 0$ into the LHS if (6) yields 0, since $\varphi_{\tau}(0, 0) = 0$. It follows that the only level of quality satisfying the IC constraint is $S = 0$. As shown in Lemma 2, the Bertrand equilibrium contract when $S = 0$ is described by (4). It is worth recalling that contract (4) is Pareto dominated by the equilibrium contract (14) arising with "less extreme" competition.

A major lesson comes from our robustness check. Firms' profits and quality level are positive at equilibrium only if $\varphi_{\tau\sigma} > 0$, i.e. only if the bigger a company is, the more sensitive the probability of sending a signal of bad quality becomes to the number of clients cheated. On the contrary, the potential harmful effect of free entry on consumer welfare is more general in that it does not depend on the sign of the cross derivative $\varphi_{\tau\sigma}$. These considerations enable us to formulate an important policy recommendation: when quality is an issue, it is not clear whether authorities should promote competition to increase consumer (and social) welfare; further investigation would be desirable.

7 Concluding Remarks

In this paper we tackled the issue of the quality of after-sales services provided by competitive insurance companies. We initially characterized the first best contract in a static context and then showed that companies have no incentive to provide a positive quality level when it is unverifiable. Finally, we considered a repeated interaction between companies and consumers in order to introduce reputation as a way to induce the former to deliver high quality. We showed that, at an equilibrium of a repeated Bertrand game among the insurers, (i) firms end up with positive profits and (ii) competition turns out to be harmful for the consumers in that it increases the companies' incentive to cheat by providing zero quality after-sales services. The former result hinges on the following reasonable assumption: as the market share of a company decreases, the probability that consumers receive a signal of bad quality becomes less sensitive to the number of clients cheated, hence the company finds it more profitable to reduce quality in order to save on costs. On the contrary, the latter finding is more general in that it holds regardless of the public signal functional form.

There are several possible extensions of the current analysis. Probably the most intuitive one is considering risk-averse insurers. In our opinion, this alternative assumption might reinforce our finding on competition and consumers' welfare, at least if we also abandon the assumption of a continuum of clients. Indeed, more competition implies smaller market share for each insurer and, in turn, lower diversification of risks. As a consequence, small risk-averse insurers turn out to be less efficient: this is likely to make the consumers worse-off.

A Appendix

A.1 Proof of Lemma 1

The proof is divided into two parts: (i) we first compute the first best contract $\left(R^{FB}, \beta^{FB}, S^{FB}\right)$; (ii) we then show that such a contract can be obtained as the outcome of Bertrand competition.

(i) The first best contract is the solution to the following problem: a representative client maximizes the expected utility subject to the firm's participation constraint and the nonnegativity conditions. In symbols:

$$\max_{R,\beta,S} pU\left(W-D+R,S\right)+\left(1-p\right)U\left(W-\beta R\right)$$
$$s.t.\ \left[\left(1-p\right)\beta-p\right]R-pc\left(S\right)\geq 0,\ S\geq 0,\ R\geq 0,\ \beta\geq 0.$$

Ignoring the nonnegativity conditions, the first order conditions with respect to R, β and S are:

$$pU_W\left(W-D+R,S\right)-\beta\left(1-p\right)U_W\left(W-\beta R\right)+\lambda\left[\left(1-p\right)\beta-p\right]=0, \tag{18}$$

$$-R\left(1-p\right)U'\left(W-\beta R\right)+\lambda\left(1-p\right)R=0, \tag{19}$$

$$pU_S\left(W-D+R,S\right)-\lambda pc'\left(S\right)=0, \tag{20}$$

respectively, where λ is the Lagrangian multiplier of the firm's participation constraint. If $R, \beta > 0$ we obtain $\lambda = U'(W - \beta R)$ from (19). As a consequence the participation constraint is binding:

$$[(1-p)\beta - p] R - pc(S) = 0 \tag{21}$$

Substituting $\lambda = U'(W - \beta R)$ into (18) and (20) gives

$$U_W(W - D + R, S) = U'(W - \beta R), \tag{22}$$

$$c'(S) = \frac{U_S(W - D + R, S)}{U'(W - \beta R)}, \tag{23}$$

respectively. Equality (23) has a solution, denoted by $S(\beta, R)$, for any level of β and R, since $c'(S)$ is increasing in S, with $c'(0) \to 0$, and U_S/U' is positive and decreasing in S, given $U_{SS} < 0$. Applying the implicit function theorem to (23) one can prove that the partial derivative of $S(\beta, R)$ with respect to β, $S_\beta(\beta, R)$, is negative. Substituting $S(\beta, R)$ into the binding constraint and solving by β one gets

$$\beta = \frac{p}{1-p} \frac{R + c(S(\beta, R))}{R}$$

which has a solution, $\beta(R)$, for any level of R, since the RHS is positive and decreasing in β given $S_\beta(\beta, R) < 0$ and $c'(S) > 0$. The implicit function theorem, applied both to $S(\beta, R)$ and $\beta(R)$, ensures that the latter is a continuous function. R may range from 0, i.e. no insurance, to the amount $R = \frac{D}{1+\beta}$ that equates consumers' monetary wealth across states, i.e. full insurance. If $R \to 0$, S must be close to zero as well, otherwise (21) cannot be satisfied given $c'(S) > 0$. Therefore the LHS of (22) reduces to $U'(W - D)$, which is greater than the RHS, $U'(W)$; on the contrary, for $R = \frac{D}{1+\beta}$ the LHS, $U_W\left(W - \frac{\beta}{1+\beta}D, S\right)$, is lower than the RHS, $U'\left(W - \frac{\beta}{1+\beta}D\right)$ since $U_{WS} < 0$. These two results, together with the fact that $\beta(R)$ is a continuous function, ensure that at least one equilibrium does exist.

(ii) We claim that the equilibrium contract when the firms compete à la Bertrand and S is verifiable has the following characteristics: (1) it is on the same indifference surface of the clients; it replicates the first best contract, i.e. (2) expected profits of the insurance companies are nought and (3) expected utility of the clients is maximum. To prove the first claim it is sufficient to notice that if the equilibrium contracts belonged to different indifference surfaces of the clients, then at least one company would get all the consumers and might be profitably deviate by slightly modifying one or more of the three contract variables, R, β or S. To prove the second claim it is sufficient to notice that if the equilibrium contracts guaranteed positive profits to the companies, then one of them would be able to profitably deviate by slightly modifying one or more among R, β or S and getting all the consumers. Also the third claim has an analogous proof. If the utility of the consumer were not maximal under the participation constraint of the insurance company, there would be ways to undercut the competitors and increase profits: a contradiction.

A.2 Proof of Lemma 2

We first compute the optimal contract when S is not verifiable as the solution to the following problem: a representative client maximizes his utility subject to the participation constraint of a representative firm and a second constraint who incorporates the fact that service quality level is zero. The client solves then the following problem:

$$\max_{R,\beta} pU(W-D+R)+(1-p)U(W-\beta R)$$
$$s.t.\ \ [(1-p)\beta - p]R \geq 0,\ R\geq 0,\ \beta \geq 0.$$

First order conditions w.r.t to R, β and the Lagrangian multiplier λ are:

$$pU'(W-D+R)-\beta(1-p)U'(W-\beta R)+\lambda[(1-p)\beta-p]=0, \tag{24}$$

$$-R(1-p)U'(W-\beta R)+\lambda(1-p)R=0, \tag{25}$$

$$[(1-p)\beta-p]R=0, \tag{26}$$

respectively. If $R>0$, equalities (25) and (26) imply

$$\lambda = U'(W-\beta R) \ \text{ and } \ (1-p)\beta = p,$$

respectively. Substituting the previous equations into (24) gives

$$U'(W-D+R)=U'(W-\beta R),$$

Solving the above equality by R and substituting $\beta=\frac{p}{1-p}$, one gets full insurance:

$$R=D(1-p).$$

To show that contract (4) can be obtained as the outcome of Bertrand competition it is sufficient to repeat part (ii) of the proof of Lemma 1, by simply replacing $S=S^{FB}$ with $S=0$.

A.3 Proof of Lemma 3

First notice that $V_{i,t}$ in (5) decreases with $c\left(S^A_{i,t}\right)$, hence insurer i sets $S^A_{i,t}=0$, i.e. she provides zero quality when cheating customers. After substituting $S^A_{i,t}=0$, we compute the derivative of $V_{i,t}$ w.r.t. $\tau_{i,t}$:

$$\frac{\partial V_{i,t}}{\partial \tau_{i,t}}=\sigma_{i,t}pc(S_{i,t})-\delta\varphi_\tau(\tau_{i,t},\sigma_{i,t})V_{i,t+1}. \tag{27}$$

If we are able to prove that $\frac{\partial V_{i,t}}{\partial \tau_{i,t}}\leq 0$ for any $\tau_{i,t}\geq 0$, then $\tau_{i,t}=0$ is an optimum for any i at any t. First notice that

$$\frac{\partial^2 V_{i,t}}{\partial \tau^2_{i,t}}=-\delta\varphi_{\tau\tau}(\tau_{i,t},\sigma_{i,t})V_{i,t+1}\leq 0 \text{ for } t=0,...,\infty.$$

Hence a sufficient (which is also necessary in case of differentiable functions) condition for the assertion is that $\frac{\partial V_{i,t}}{\partial \tau_{i,t}}\leq 0$ at $\tau_{i,t}=0$. We assume that the solution is stationary, i.e. $V_{i,t}=V_{i,t+1}$ for all i and all $t=0,...\infty$, and we then check whether such a solution is admissible. Putting $V_{i,t}=V_{i,t+1}$ in (5) and omitting subscript i one gets

$$V=\sigma\frac{\tilde{\Pi}+p\tau c(S)}{1-\delta(1-\varphi)}.$$

Substituting the above value into (27) and omitting subscripts i and t one gets

$$\sigma pc(S) - \delta\varphi_\tau \sigma \frac{\tilde{\Pi} + p\tau c(S)}{1-\delta(1-\varphi)}.$$

This expression is nonpositive in $\tau = 0$ if and only if

$$pc(S) - \delta\varphi_\tau(0,\sigma)\frac{\tilde{\Pi}}{1-\delta} \leq 0.$$

Rearranging

$$\varphi_\tau(0,\sigma) \geq \frac{1-\delta}{\delta}\frac{pc(S)}{\tilde{\Pi}}.$$

A.4 Proof of Lemma 4

We solve the problem of a representative client maximizing his single-period utility subject to a representative firm's participation and IC constraints. Notice that the former, $\tilde{\Pi} \geq 0$, is slack because it is implied by the latter: see (7). The problem is hence as follows:

$$\begin{aligned} &\max_{R,\beta,S,\sigma} pU(W-D+R,S) + (1-p)U(W-\beta R) \\ &\text{s.t. } p(1-\delta)c(S) - \delta\varphi_\tau(0,\sigma)\tilde{\Pi} \leq 0, \\ &S \geq 0, \quad R \geq 0, \quad \beta \geq 0, \quad \sigma \in (0,1]. \end{aligned} \tag{28}$$

where the first constraint is (6) after rearrangement. Considering only the IC constraint, FOCs w.r.t. R, β, S and σ are respectively:

$$pU_W(W-D+R,S) - \beta(1-p)U'(W-\beta R) + \lambda\delta\varphi_\tau(0,\sigma)[(1-p)\beta - p] = 0 \tag{29}$$

$$-R(1-p)U'(W-\beta R) + \lambda\delta\varphi_\tau(0,\sigma)(1-p)R = 0, \tag{30}$$

$$pU_S(W-D+R,S) - \lambda pc'(S)(1-\delta+\delta\varphi_\tau(0,\sigma)) = 0, \tag{31}$$

$$\lambda\delta\varphi_{\tau\sigma}(0,\sigma)\{[(1-p)\beta - p]R - pc(S)\} = 0. \tag{32}$$

Equation (30) yields:

$$\lambda = \frac{U'(W-\beta R)}{\delta\varphi_\tau(0,\sigma)}, \tag{33}$$

which substituted into (29) gives

$$U_W(W-D+R,S) = U'(W-\beta R). \tag{34}$$

This is the condition of optimal insurance, that is, the marginal utility of money should be the same in all states. Substituting (33) and (34) into (31) yields

$$c'(S) = \frac{\delta\varphi_\tau(0,\sigma)}{1-\delta+\delta\varphi_\tau(0,\sigma)}\frac{U_S(W-D+R,S)}{U'(W-\beta R)}.$$

From the constraint (28) taken with equality we obtain:

$$[(1-p)\beta - p]R = pc(S)\left(\frac{1-\delta}{\delta\varphi_\tau(0,\sigma)} + 1\right)$$

which substituted in (32) yields:

$$U'(W-\beta R)\frac{(1-\delta)\,pc(S)}{\delta\varphi_\tau(0,\sigma)}=0;$$

this is never possible since the LHS is positive for any $S>0$; hence the first order condition w.r.t. σ is always positive. This implies that the optimal σ equals 1. Finally, from the constraint taken with equality we compute β:

$$\beta=\frac{p}{1-p}+\frac{pc(S)}{(1-p)\,R}\frac{1-\delta+\delta\varphi_\tau(0,\sigma)}{\delta\varphi_\tau(0,\sigma)}.$$

Rearranging the above expression yields the result. Relying on the same reasoning used in the proof of Lemma 1 and on the conditions specified there ensures that system (8) has at least a solution.

A.5 Proof of Lemma 5

By setting

$$\begin{aligned} W_b &= W-D+R, \\ W_g &= W-\beta R, \end{aligned}$$

the problem (28) can be transformed into the equivalent:

$$\begin{gathered} \max_{W_g,W_b,S} pU(W_b,S)+(1-p)\,U(W_g) \\ \text{s.t. } p(1-\delta)\,c(S)-\delta\varphi_\tau(0,\sigma)\,\tilde{\Pi}\leq 0, \\ \tilde{\Pi}=W-W_g(1-p)-p(W_b+D)-pc(S)\,. \end{gathered} \tag{35}$$

First order conditions are:

$$\begin{cases} (1-p)\,U'(W_g)-\lambda\delta\varphi_\tau(0,\sigma)\,(1-p)=0 \\ pU_W(W_b,S)-\lambda\delta\varphi_\tau(0,\sigma)\,p=0 \\ pU_S(W_b,S)-\lambda pc'(S)\,(1-\delta+\delta\varphi_\tau(0,\sigma))=0 \\ -p(1-\delta)\,c(S)+\delta\varphi_\tau(0,\sigma)\,(W-W_g(1-p)-p(W_b+D)-pc(S))=0. \end{cases} \tag{36}$$

The bordered Hessian of the problem is:

$$H=\begin{bmatrix} (1-p)\,U'' & 0 & 0 & -\delta\varphi_\tau(1-p) \\ 0 & pU_{WW} & pU_{WS} & -\delta\varphi_\tau p \\ 0 & pU_{SW} & pU_{SS}-\lambda pc''(1-\delta+\delta\varphi_\tau) & -pc'(1-\delta+\delta\varphi_\tau) \\ -\delta\varphi_\tau(1-p) & -\delta\varphi_\tau p & -pc'(1-\delta+\delta\varphi_\tau) & 0 \end{bmatrix}$$

According to the implicit function theorem $\frac{\partial S}{\partial\sigma}=\frac{\det(H_\sigma)}{\det(H)}$, where H_σ is a matrix obtained from H by substituting the third column with the following vector

$$\begin{bmatrix} \lambda\delta\varphi_{\tau\sigma}(1-p) \\ \lambda\delta\varphi_{\tau\sigma}p \\ \lambda pc'\delta\varphi_{\tau\sigma} \\ -\delta\varphi_{\tau\sigma}(W-W_g(1-p)-p(W_b+D)-pc(S)) \end{bmatrix}$$

The above vector, in turn, derives from differentiating the system of FOCs with respect to σ and changing the sign. The second order conditions of the objective function and the constraint of problem (35) ensure that the determinant of above matrix H, $\det(H)$, is negative. It follows that $sign\frac{\partial S}{\partial \sigma} = -sign\det(H_\sigma)$. By direct computation one gets

$$\frac{\det(H_\sigma)}{\delta p^2 \varphi_{\tau\sigma}(1-p)} = \varphi_\tau \lambda \delta c'(1-\delta)\left[(1-p)U_{WW} + pU''\right] + U''\tilde{\Pi}\left[U_{WS}\varphi_\tau\delta - (1-\delta+\delta\varphi_\tau)U_{WW}c'\right]. \quad (37)$$

Since $\delta p^2\varphi_{\tau\sigma}(1-p) > 0$, then $sign\det(H_\sigma) = sign(37)$. From (36) we have $\frac{U_W}{\lambda} = \delta\varphi_\tau$, $\frac{U_S}{\lambda} = c'(1-\delta+\delta\varphi_\tau)$ and $\delta\varphi_\tau\tilde{\Pi} = p(1-\delta)c(S)$. Plugging these values into (37) yields

$$U_W c'(1-\delta)\left[(1-p)U_{WW} + pU''\right] + U''\frac{1}{\lambda}\frac{p}{\delta\varphi_\tau}(1-\delta)c(U_{WS}U_W - U_S U_{WW}).$$

Substituting again $\frac{U_W}{\lambda} = \delta\varphi_\tau$ and dividing everything by $U_W c'(1-\delta)$ we obtain

$$(1-p)U_{WW} + pU'' + p\frac{c}{c'}\frac{U''}{U_W}U_{WS} - p\frac{c}{c'}\frac{U''}{U_W}\frac{U_S}{U_W}U_{WW}. \quad (38)$$

If the above expression is negative (positive), then $\det(H_\sigma) < (>)\,0$ and $\frac{\partial S}{\partial \sigma} > (<)\,0$. Rearranging inequalities $(38) < (>)\,0$ yields conditions (9) and (10), respectively.

A.6 Proof of Proposition 1

We solve the game backwards and start from point 4.

(i) We separate the two cases: $n \leq n_k$ and $n > n_k$. In the first case, contract (14) is equivalent to contract (8) with the only exception of a different market share. As a consequence, contract (14) satisfies with the equality the IC constraint (6): the clients accept it since it is the maximum they can get. To prove it, note that two possible deviations are available to any insurer i: offering a different contract with either (a) better or (b) worse conditions for the clients. In the subcase (a), each consumer, according to the equilibrium strategy, expects all the other clients to refuse this contract. In such a case, insurer i's market share is nil, $\sigma_i = 0$, with the effect that the contract does not satisfy her IC constraint. Indeed, the LHS of (6) $\varphi_\tau(0,0)$ is nil by assumption, while the RHS is positive (and nil only for $S = 0$). The consumers anticipate that $S_i = 0$ and prefer thus to turn to any other competitor. In the subcase (b) the contract satisfies strictly the IC constraint (6). Yet, no clients would accept it because any other insurance company offers better contractual terms.

Consider now the case of $n > n_k$. If all firms offer the contract (15), the clients accept it since, given $S = 0$, it is the maximum they can get. Again, two possible deviations are available to any insurer i. We focus on that ensuring a higher utility to the clients, since the discussion of the other one is trivial. Such a contract must promise a positive quality level, $S_i > 0$, otherwise the firm's participation constraint would be violated. Yet, each client expects that no other client will accept the contract. Therefore the market share of insurer i would be nought, $\sigma = 0$, and the contract would not satisfy her IC constraint. As a consequence, the consumers anticipate that $S_i = 0$ and prefer thus to turn to any other competitor.

(ii) Let us now turn to points 2 and 3. We first prove that in each period $t \geq 0$ the contract (14) is an equilibrium one for any $n_k \geq n \geq 2$. To this aim, first recall that the contract characterized in (14) satisfies the IC constraint (6) with equality: if all the firms offer this contract the consumers accept it, hence the former get $\tilde{\Pi}\left(\hat{\beta}(n), \hat{R}(n), \hat{S}(n)\right) > 0$ on each contract stipulated at each time t. Given that we proved that any other contract would be refused by consumer there is no profitable deviation, because it would bring zero profits instead of positive ones. Now suppose that $n > n_k$. In this case the consumers accept only the contract (15), which yields zero profits; any other contract would also bring zero profits, hence there is no strictly profitable deviation.

(iii) We finally discuss point 1 by treating separately two subcases: (a) quality level S^r is positively affected by market share σ, $\frac{\partial S^r}{\partial \sigma} > 0$; (b) quality level S^r is negatively affected by market share σ, $\frac{\partial S^r}{\partial \sigma} < 0$.

In case (a) $\hat{n}$ is the equilibrium number of firms. To see this, suppose first $n < \hat{n}$ firms enter. It follows that $\hat{S}(n+1) > \bar{S}$. This means that at least an additional firm can enter and offer contract (14) with $n+1$ firms. Such an offer would be accepted given the consumer implicit expectations (16), which we discuss in the text. Therefore the entrant would end up with positive profits. We conclude that $n < \hat{n}$ cannot be an equilibrium of the initial entry stage. Now suppose $n > \hat{n}$ firms enter. This implies $\hat{S}(n) < \bar{S}$, hence given (16) no client believes that the entrant and the incumbents will provide $S > 0$. It follows that the firms compete à la Bertrand and offer a contract which entails the maximum expected utility for the clients with $S = 0$ and zero expected profits for the firms. Therefore, entry is not a strictly profitable strategy for an outside firm. We are able to conclude that the only equilibrium number of firms is $n = \hat{n}$.

In case (b) $\breve{n}$ is the equilibrium number of firms. To prove it, suppose first $n < \breve{n}$ firms enter. As showed in the text (see the discussion of Proposition 1), $\tilde{U}(n)$ is decreasing in n. It follows that $\tilde{U}(n) > pU(W-D) + (1-p)U(W)$. This means that at least an additional firm can enter and offer contract (14) with $n+1$ firms. Such an offer would be accepted given that the consumers participation constraint is not active, hence the entrant would end up with positive profits: $n < \breve{n}$ cannot be an equilibrium of the initial entry stage. Now suppose $n > \breve{n}$ firms enter. This implies $\tilde{U}(n) < pU(W-D) + (1-p)U(W)$, hence no client will accept the contract: entry is not a strictly profitable strategy for an outside firm. We are able to conclude that the only equilibrium number of firms is $n = \breve{n}$.

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